

* تمرینات فصل دوم (معادلات با مشتقات جزئی)

شماره ۵.۲ صفحه ۹۱۱
مسئله ارتعاش را در حالتی حل کنید که اندام در روی مرز ناحیه صفر و سرعت اولیه آن به صورت داده شده باشد.

a. $u(x,0) = x(1-x)$ $0 \leq x \leq 1$
 $u_t(x,0) = 0$

$\Leftrightarrow \lambda_n = c n \pi$ و $L=1$ در این سؤال

$$a_n = \int_0^1 x(1-x) \sin n\pi x \, dx$$

$$a_n = \int_0^1 \left[(x-x^2) \left(\frac{-\cos n\pi x}{n\pi} \right) - (1-2x) \left(\frac{-\sin n\pi x}{(n\pi)^2} \right) + (-2) \frac{\cos n\pi x}{(n\pi)^3} \right] dx$$

$a_n = \frac{2}{(n\pi)^3} [1 + (-1)^{n+1}]$ $u_t(x,0) = 0 \Rightarrow b_n = 0$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2}{(n\pi)^3} [1 + (-1)^{n+1}] \cos c n \pi t \sin n\pi x$$

b. $u(x,0) = 3 \sin x$, $u_t(x,0) = 0$ $0 \leq x \leq \pi$

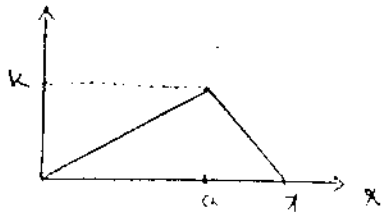
با توجه به $u(x,0) = 3 \sin x$ نتیجه می‌گیریم سبب سینوسی مورد نیاز این تابع در برابر a_n خواهد بود. بنابراین جمله اول سبب ضرب $a_1 = 3$ می‌باشد پس سایر جمله‌ها صفر رانده و داریم:

$$\lambda_1 = \frac{c\pi}{\pi} = c$$

$$\Rightarrow u(x,t) = a_1 \sin x \cos ct = 3 \cos ct \sin x$$

۲- مسئله ارتعاش نخ را در حالتی حل کنید که اندام اولیه ارتعاش را در یک ربع زیر و سرعت اولیه آن صفر باشد. وضعیت ارتعاش را در حین ناله که خواستگاه خواهد نمود به کمک جواب حاصل از روش دالامبر رسم کنید. ($c=1$)

الف)



$$u(x, 0) = \begin{cases} \frac{k}{a} x & 0 \leq x \leq a \\ \frac{k}{a-x} (\pi - x) & a < x \leq \pi \end{cases}$$

$$u(0, t) = 0 \quad u(a, t) = 0 \quad u(\pi, t) = 0$$

$$c = 1$$

$$L = \pi$$

$$\lambda_n = \frac{cn\pi}{L} = n$$

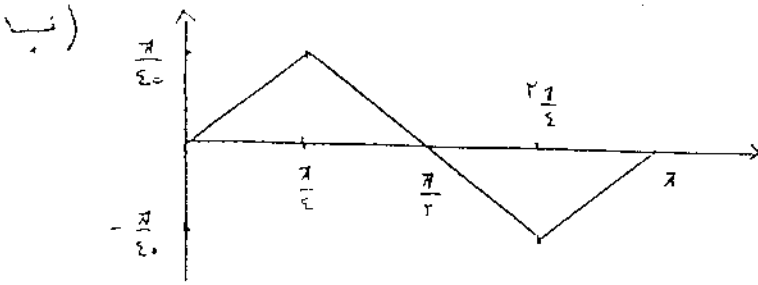
$$b_n = 0$$

$$a_n = \frac{2}{\pi} \int_0^a \frac{k}{a} x \sin nx \, dx + \frac{2}{\pi} \int_a^\pi \frac{k}{a-x} (\pi - x) \sin nx \, dx$$

$$= \frac{2}{\pi} \left[\frac{k}{a} x \left(\frac{-\cos nx}{n} \right) - \frac{k}{a} \left(\frac{-\sin nx}{n^2} \right) \right]_0^a + \frac{2}{\pi} \left[\frac{k(\pi-x)}{a-x} \left(\frac{-\cos nx}{n} \right) - \frac{k(-\sin nx)}{(a-x)n^2} \right]_a^\pi$$

$$= \frac{2k}{a(a-x)} \left[-\frac{\pi \sin na}{n^2} \right] = \frac{2k}{a(a-x)} \frac{\sin na}{n^2} \Rightarrow$$

$$u(x, t) = \frac{2k}{a(a-x)} \sum_{n=1}^{\infty} \frac{\sin na}{n^2} \cos nt \sin nx$$



$$u(0,t) = u(\pi,t) = 0$$

$$u_t(x,0) = 0$$

$$u(x,0) = f(x)$$

$$f(x) = \begin{cases} \frac{1}{l} x & 0 \leq x < \frac{\pi}{2} \\ -\frac{1}{l} (x - \frac{\pi}{2}) & \frac{\pi}{2} \leq x < \frac{3\pi}{2} \\ \frac{1}{l} (x - \pi) & \frac{3\pi}{2} \leq x < \pi \end{cases}$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \frac{1}{l} x \sin nx \, dx + \frac{1}{\pi} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\frac{1}{l} (x - \frac{\pi}{2}) \sin nx \, dx + \frac{1}{\pi} \int_{\frac{3\pi}{2}}^{\pi} \frac{1}{l} (x - \pi) \sin nx \, dx \\ &= \frac{1}{\pi} \left[-\frac{1}{l} x \frac{\cos nx}{n} + \frac{1}{l} \frac{\sin nx}{n^2} \right]_0^{\frac{\pi}{2}} + \frac{1}{\pi} \left[\frac{1}{l} (x - \frac{\pi}{2}) \frac{\cos nx}{n} - \frac{1}{l} \frac{\sin nx}{n^2} \right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \\ &\quad + \left[-\frac{1}{l} (x - \pi) \frac{\cos nx}{n} + \frac{1}{l} \frac{\sin nx}{n^2} \right]_{\frac{3\pi}{2}}^{\pi} = \frac{2}{l \cdot \pi} \left[\frac{\sin \frac{n\pi}{2}}{n^2} - \frac{\sin \frac{3n\pi}{2}}{n^2} \right] \end{aligned}$$

$$a_1 = 0 \quad a_2 = \frac{2}{\pi l} \times \frac{1}{2} \quad a_3 = 0 \quad a_4 = \frac{2}{\pi l} \times 0 = 0 \quad a_5 = 0 \quad a_6 = \frac{2}{\pi l} \times \left(-\frac{1}{4}\right)$$

$$a_7 = 0 \quad a_8 = 0 \quad a_9 = 0 \quad a_{10} = \frac{2}{\pi l} \times \frac{1}{10}$$

$$\Rightarrow u = \frac{2}{\pi l} \left(\frac{1}{2} \cos 2t \sin 2x - \frac{1}{4} \cos 4t \sin 4x + \frac{1}{10} \cos 10t \sin 10x + \dots \right)$$

-۳- جبراساتريك ارسالانف زيرل برورنن قورنل دت اورور

a. $u_x + u_y = 0$

$u = g(x) f(y)$

$u_x = f(y) g'(x)$

$u_y = g(x) f'(y)$

$u_x + u_y = g'(x) f(y) + g(x) f'(y) = 0 \Rightarrow g'(x) f(y) = -g(x) f'(y) \Rightarrow \frac{g'(x)}{g(x)} = \frac{f'(y)}{f(y)} = k$

$f(y) + c_0 f'(y) = 0 \Rightarrow f(y) = A e^{-\frac{1}{c_0} y}$

تاير برابر اعدنات باستد

$g(x) - c_0 g'(x) \Rightarrow g(x) = A e^{\frac{1}{c_0} x}$

$\Rightarrow u = g(x) f(y) = AA' e^{\frac{1}{c_0} (x-y)}$

$AA' = k$
 $\frac{1}{c_0} = c$
 $k e^{c(x-y)}$

b. $y u_x = \alpha u_y$

$u = f(x) g(y)$

$u_x = f'(x) g(y)$ و $u_y = g'(y) f(x)$

$y g(y) f'(x) = \alpha f(x) g'(y)$

$\frac{y g(y)}{g'(y)} = \frac{\alpha f(x)}{f'(x)} = k$

$\Rightarrow y(g(y)) - k g'(y) = 0$ ①

$\alpha f(x) - k f'(x) = 0$ ②

① $\Rightarrow \frac{g'(y)}{g(y)} = \frac{y}{k} \Rightarrow \ln g(y) = \frac{1}{rk} y^r \Rightarrow g(y) = e^{\frac{y^r}{rk}}$

② $\Rightarrow \alpha f(x) = k f'(x) \Rightarrow \frac{f'(x)}{f(x)} = \frac{\alpha}{k} \Rightarrow \ln f(x) = \frac{\alpha^r}{rk} x^r \Rightarrow f(x) = e^{\frac{\alpha^r}{rk} x^r}$

$u = e^{\frac{1}{rk} (\alpha^r x^r + y^r)}$

$u = c$

c. $xy'' = yu_y$

$u_x = f'(x)g(y)$ $u_y = g'(y)f(x)$

$\Rightarrow x \frac{f'(x)}{f(x)} = \frac{y g'(y)}{g(y)} = k \Rightarrow x f'(x) - k f(x) = 0$ (1)

$y g'(y) - k g(y) = 0$ (2)

(1) $\Rightarrow x f'(x) = k f(x) \Rightarrow \frac{f'(x)}{f(x)} = \frac{k}{x}$ ^{انتگرال گیری} $\Rightarrow \int \frac{f'(x)}{f(x)} = \int \frac{k}{x} \Rightarrow \ln f(x) = k \ln x + \ln c_0$

$\Rightarrow f(x) = c_0 x^k$ (1')

(2) $\Rightarrow y g'(y) = k g(y) \Rightarrow \frac{g'(y)}{g(y)} = \frac{k}{y} \Rightarrow \int \frac{g'(y)}{g(y)} = \int \frac{k}{y} \Rightarrow \ln g(y) = k \ln y + \ln c_1$

$g(y) = c_1 y^k$ (2')

(1), (2) $\Rightarrow u = c_0 c_1 x^k y^k$

d. $u_{xy} = u$

$u_x = f'(x)g(y)$ $u_y = g'(y)f(x)$

$\Rightarrow f'(x)g'(y) = f(x)g(y) \Rightarrow \frac{f'(x)}{f(x)} = \frac{g'(y)}{g(y)} = k$

$f'(x) = k f(x) \Rightarrow f(x) = e^{kx}$

$g'(y) = k g(y) \Rightarrow g(y) = e^{\frac{y}{k}}$

$\Rightarrow u = e^{k(x+y)}$

e. $u_{xx} + u_x - ru = 0$

$D = \frac{du}{dx} \Rightarrow (D^2 + D - r)u = 0 \Rightarrow D = 1, D = -r$

$u = A e^x + B e^{-rx}$ (1)

در (1) A و B توابعی از x هستند.

$u = f(x) e^x + g(x) e^{-rx}$

خواهیم داشت:

۳- درک از عبارات و با استفاده از روشی که در ادامه خواهیم دید.

a) $u_{xx} + u_{yy} = 1$, $v = x+y$, $z = x-y$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial v} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial z} \frac{\partial z}{\partial x} = \frac{\partial u}{\partial v} + \frac{\partial u}{\partial z}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial v^2} \frac{\partial v}{\partial x} + \frac{\partial^2 u}{\partial z^2} \frac{\partial z}{\partial x} + 2 \left(\frac{\partial^2 u}{\partial v \partial z} \frac{\partial v}{\partial x} + \frac{\partial^2 u}{\partial z \partial v} \frac{\partial z}{\partial x} \right)$$

$$= \frac{\partial^2 u}{\partial v^2} + 2 \frac{\partial^2 u}{\partial v \partial z} + \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial v} \frac{\partial v}{\partial y} + \frac{\partial u}{\partial z} \frac{\partial z}{\partial y} = \frac{\partial u}{\partial v} - \frac{\partial u}{\partial z}$$

$$\left(\frac{\partial^2 u}{\partial v^2} + \frac{\partial^2 u}{\partial z^2} + 2 \frac{\partial^2 u}{\partial v \partial z} \right) + \left(\frac{\partial^2 u}{\partial v^2} - \frac{\partial^2 u}{\partial z^2} - 2 \frac{\partial^2 u}{\partial v \partial z} \right) = \frac{\partial^2 u}{\partial v^2} - \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial^2 u}{\partial v^2} = \frac{\partial^2 u}{\partial v^2} + \frac{\partial^2 u}{\partial z^2} + 1 \Rightarrow \frac{\partial^2 u}{\partial z^2} = 1$$

$$= \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial v^2} = 1$$

با استفاده از روش جداسازی متغیرها:

$$\frac{\partial^2 u}{\partial z^2} = 1 \Rightarrow \frac{\partial u}{\partial z} = z + C_1$$

$$\frac{\partial u}{\partial z} = z + C_1 \Rightarrow u = \frac{z^2}{2} + C_1 z + C_2$$

$$\Rightarrow u = \int h(z) dz + \phi(v) \Rightarrow$$

$$u = \phi(z) + \psi(v) = \phi(x-y) + \psi(x+y)$$

b) $u_{xx} - 2u_{xy} + u_{yy} = 0$ $v = y$ $z = x + y$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial v} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial z} \frac{\partial z}{\partial x} = \frac{\partial u}{\partial z}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial v^2} \frac{\partial v}{\partial x} + \frac{\partial^2 u}{\partial z^2} \frac{\partial z}{\partial x} = \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial v \partial z} \frac{\partial v}{\partial x} + \frac{\partial^2 u}{\partial z^2} \frac{\partial z}{\partial x} = \frac{\partial^2 u}{\partial v \partial z} + \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial v} \frac{\partial v}{\partial y} + \frac{\partial u}{\partial z} \frac{\partial z}{\partial y} = \frac{\partial u}{\partial v} + \frac{\partial u}{\partial z}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial v^2} \frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial z^2} \frac{\partial z}{\partial y} + \frac{\partial^2 u}{\partial v \partial z} \frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial v \partial z} \frac{\partial z}{\partial y}$$

$$= \frac{\partial^2 u}{\partial v^2} + \frac{\partial^2 u}{\partial z^2} + 2 \frac{\partial^2 u}{\partial v \partial z}$$

با این فرضیات:

$$\frac{\partial^2 u}{\partial z^2} - 2 \frac{\partial^2 u}{\partial v \partial z} - \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial v^2} + \frac{\partial^2 u}{\partial z^2} + 2 \frac{\partial^2 u}{\partial v \partial z} + \frac{\partial^2 u}{\partial z^2} = 0$$

$$\Rightarrow \frac{\partial^2 u}{\partial v^2} = 0 \Rightarrow \frac{\partial u}{\partial v} = h(z) \Rightarrow u = \int h(z) dv + \Phi(z)$$

$$u = v h(z) + \Phi(z) = y h(x+y) + \Phi(x+y)$$

C) $u_{xy} = x u_{xz} + u_x$ $v = y$ $z = xy$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial z} \frac{\partial z}{\partial x} + \frac{\partial u}{\partial v} \frac{\partial v}{\partial x} = \frac{\partial u}{\partial z} \cdot y + \frac{\partial u}{\partial v} \cdot 0 = y \frac{\partial u}{\partial z}$$

$$\frac{\partial^2 u}{\partial x^2} = y \left(\frac{\partial^2 u}{\partial z^2} \frac{\partial z}{\partial x} + \frac{\partial^2 u}{\partial z \partial v} \frac{\partial v}{\partial x} \right) = y \left(\frac{\partial^2 u}{\partial z^2} \cdot y + \frac{\partial^2 u}{\partial z \partial v} \cdot 0 \right) = y^2 \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial u}{\partial x \partial y} = \frac{\partial u}{\partial z} + y \left(\frac{\partial^2 u}{\partial z^2} \frac{\partial z}{\partial y} + \frac{\partial^2 u}{\partial z \partial v} \frac{\partial v}{\partial y} \right) = \frac{\partial u}{\partial z} + y \left(\frac{\partial^2 u}{\partial z^2} \cdot x + \frac{\partial^2 u}{\partial z \partial v} \cdot 1 \right) = \frac{\partial u}{\partial z} + xy \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial z \partial v}$$

$$y \frac{\partial u}{\partial z} + y^2 \frac{\partial^2 u}{\partial z^2} + xy \frac{\partial^2 u}{\partial z^2} = xy^2 \frac{\partial^2 u}{\partial z^2} + y \frac{\partial u}{\partial z} \quad \text{: persamaan kedua}$$

$$\Rightarrow y^2 \frac{\partial^2 u}{\partial z^2} = 0 \Rightarrow \frac{\partial^2 u}{\partial z^2} = 0 \quad \frac{\partial u}{\partial z} = h(z)$$

$$u = \int h(z) dz + \Phi(v) = \varphi(z) + \Phi(v) = \varphi(xy) + \Phi(y)$$

درجه حرارت در طول یک میله به طول ۱.۰م با یک درجه صفر در یک سر میله صفر بوده و درجه حرارت در آن سر میله برابر با ۱۰۰ درجه فارنهایت است (C=۱.۷۵۲)

a) $u(x,0) = \sin \frac{\pi}{l} x$

$$u(x,0) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x = \sum_{n=1}^{\infty} b_n \sin \frac{\pi}{l} x$$

$$\Rightarrow b_1 = 1, b_n = 0 \quad (n > 1)$$

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x e^{-\lambda_n^2 t} = \sin \frac{\pi}{l} x e^{-\frac{1}{4} \frac{\pi^2}{l^2} t}$$

b) $f(x) = \begin{cases} x & ; 0 < x < \frac{l}{2} \\ l-x & ; \frac{l}{2} < x < l \end{cases}$ $\lambda_n = \frac{1}{l} \sqrt{\Delta^2} \frac{\pi^2}{l^2}$

$$b_n = \frac{2}{l} \int_0^{\frac{l}{2}} x \sin \frac{n\pi}{l} x dx + \frac{2}{l} \int_{\frac{l}{2}}^l (l-x) \sin \frac{n\pi}{l} x dx$$

$$= \frac{2}{l} \left[-x \left(\frac{l}{n\pi} \right) \cos \frac{n\pi}{l} x + \left(\frac{l}{n\pi} \right)^2 \sin \frac{n\pi}{l} x \right]_0^{\frac{l}{2}} +$$

$$\frac{2}{l} \left[-(l-x) \cos \frac{n\pi}{l} x \left(\frac{l}{n\pi} \right) - \left(\frac{l}{n\pi} \right)^2 \sin \frac{n\pi}{l} x \right]_{\frac{l}{2}}^l = \frac{2}{l} \frac{\sin \frac{n\pi}{l} x}{n^2}$$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2}{l} \frac{\sin \frac{n\pi}{l} x}{n^2} \sin \frac{n\pi}{l} x e^{-c/l^2 \Delta^2 (n\pi)^2 t}$$

c) $f(x) = \begin{cases} x & ; -\frac{\Delta}{2} < x < \frac{\Delta}{2} \\ \Delta - x & ; \frac{\Delta}{2} < x < \Delta \end{cases}$

$$b_n = \frac{2}{\Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} x \sin \frac{n\pi}{\Delta} x dx + \frac{2}{\Delta} \int_{\frac{\Delta}{2}}^{\Delta} (\Delta - x) \sin \frac{n\pi}{\Delta} x dx$$

* ایا که در اینجا سوال است (به معنی دانستن)

۲- هر یک از مسائل زیر را حل کنید

a)

$$u_t - \sum u_{xx} = xt \quad ; \quad 0 < x < 1 \quad - \quad t > 0$$

$$u(x, 0) = \sin \pi x \quad 0 \leq x \leq 1$$

$$u(0, t) = t \quad ; \quad u(1, t) = t^2 \quad t > 0$$

$$u = v + w \quad w = at + b \rightarrow \begin{cases} u(0, t) = t = b \Rightarrow b = t \\ u(1, t) = a + b = t^2 \Rightarrow a = t^2 - t \end{cases}$$

$$v(0, t) = 0$$

$$v(1, t) = 0$$

$$v(x, 0) = \sin \pi x \quad w = (t^2 - t)x + t$$

$$u = v + w \Rightarrow v_t - \sum v_{xx} = u - u_t - 1 \quad (*)$$

$$v(x, t) = \sum_{n=1}^{\infty} G(t) \sin n\pi x \quad | \quad * \Rightarrow \sum_{n=1}^{\infty} \underbrace{[G'(t) + \varepsilon n^2 \pi^2 G(t)]}_{\gamma(t)} \sin n\pi x = u - u_t - 1$$

$$\gamma(t) = \int_0^1 (u - u_t - 1) \sin n\pi x \, dx = \int_0^1 \left[(1-t) \frac{(-1)^n}{n\pi} + \frac{(-1)^n}{n\pi} - \frac{1}{n\pi} \right]$$

$$= \int_0^1 \left[\frac{\pi(-1)^n}{n\pi} + \frac{t(-1)^{n+1}}{n\pi} - \frac{1}{n\pi} \right] = \frac{\varepsilon(-1)^n}{n\pi} + \frac{\varepsilon t(-1)^{n+1}}{n\pi} - \frac{\varepsilon}{n\pi}$$

$$\gamma(t) = G'(t) + \varepsilon n^2 \pi^2 G(t) \quad G_k = c_k e^{-\varepsilon n^2 \pi^2 t}$$

$$D + \varepsilon n^2 \pi^2 = 0 \Rightarrow D = -\varepsilon n^2 \pi^2 \quad G_p = c_p t + c_q$$

$$\rightarrow c_p + \varepsilon n^2 \pi^2 (c_p t + c_q) = \frac{\varepsilon(-1)^n}{n\pi} + \frac{(-1)^{n+1}}{n\pi} t - \frac{\varepsilon}{n\pi}$$

$$\Rightarrow \varepsilon n^2 \pi^2 c_p = \frac{\varepsilon(-1)^{n+1}}{n\pi} \Rightarrow c_p = \frac{\varepsilon(-1)^{n+1}}{\varepsilon n^2 \pi^2}$$

$$c_p + \varepsilon n^2 \pi^2 c_q = \frac{\varepsilon(-1)^n}{n\pi} - \frac{\varepsilon}{n\pi} \Rightarrow c_q = \frac{1}{\varepsilon n^2 \pi^2} \left[\frac{\varepsilon(-1)^n}{n\pi} - \frac{\varepsilon}{n\pi} + \frac{\varepsilon(-1)^n}{\varepsilon n^2 \pi^2} \right]$$

$$G(t) = c_p e^{-\varepsilon n^2 \pi^2 t} + c_q t + c_r \Rightarrow v(x, t) = \sum_{n=1}^{\infty} \left[c_p e^{-\varepsilon n^2 \pi^2 t} + c_q t + c_r \right] \sin n\pi x$$

۱۔ مسئلہ کو ماوریکوئی ناگہانی راجل لیدر صورت دے کر ان اڈے درمیلے برابر ہی ازواج زیر باشد

$$a) f(x) = \begin{cases} x & ; -r < x < r \\ 0 & ; x > r \end{cases}, f(-x) = f(x)$$

$$A(\omega) = \frac{r}{\pi} \int_0^{\infty} f(x) \cos \omega x dx = \frac{r}{\pi} \int_0^r x \cos \omega x dx$$

$$= \frac{r}{\pi} \left[\frac{x}{\omega} \sin \omega x + \frac{\cos \omega x}{\omega^2} \right]_0^r = \frac{r}{\pi} \left[\frac{r}{\omega} \sin r\omega + \frac{\cos r\omega}{\omega^2} - \frac{\cos 0}{\omega^2} \right]$$

$$\Rightarrow A(\omega) = \frac{r}{\pi \omega} \left[r \sin r\omega + \frac{\cos r\omega - 1}{\omega} \right]$$

$$u(x,t) = \int_0^{\infty} \frac{r}{\pi \omega} \left[r \sin r\omega + \frac{\cos r\omega - 1}{\omega} \right] \cos \omega x d\omega$$

(تشریح)

$$B(\omega) = \frac{r}{\pi} \int_0^{\infty} f(x) \sin \omega x dx = \frac{r}{\pi} \int_0^r x \sin \omega x dx + \frac{r}{\pi} \int_r^{\infty} r \sin \omega x dx$$

$$= \frac{r}{\pi} \left[-\frac{rx}{\omega} \cos \omega x + \frac{r}{\omega^2} \sin \omega x \right]_0^r + \frac{r}{\pi} \left[-\frac{rx}{\omega} \cos \omega x + \frac{rx}{\omega^2} \sin \omega x + \frac{r}{\omega^2} \cos \omega x \right]_r^{\infty}$$

$$= \frac{r}{\pi} \left[-\frac{r}{\omega} \cos r\omega + \frac{r}{\omega^2} \sin r\omega \right] + \frac{r}{\pi} \left[-\frac{1}{\omega} \cos r\omega + \frac{1}{\omega^2} \sin r\omega - \frac{2}{\omega^2} \cos r\omega + \frac{r}{\omega} \cos r\omega - \frac{r}{\omega^2} \sin r\omega + \frac{2}{\omega^2} \cos r\omega \right]$$

$$= \frac{r}{\pi} \left[-\frac{r}{\omega^2} \sin r\omega - \frac{1}{\omega} \cos r\omega + \frac{1}{\omega^2} \sin r\omega - \frac{2}{\omega^2} \cos r\omega + \frac{r}{\omega^2} \cos r\omega \right]$$

$$u(x,t) = \int_0^{\infty} B(\omega) \sin \omega x d\omega$$